
Raindrops keep falling on my head

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Rainfall is the cause of many problems, not least flooding, the cause of widespread destruction over northern Europe earlier this year. To most people, though, having to run that final 100 m from the bus stop to the house to avoid

being soaked by pouring rain is often of more immediate concern. But does running in the rain actually reduce how wet we get? This problem is one that many people will have thought about at some time, and quite recently

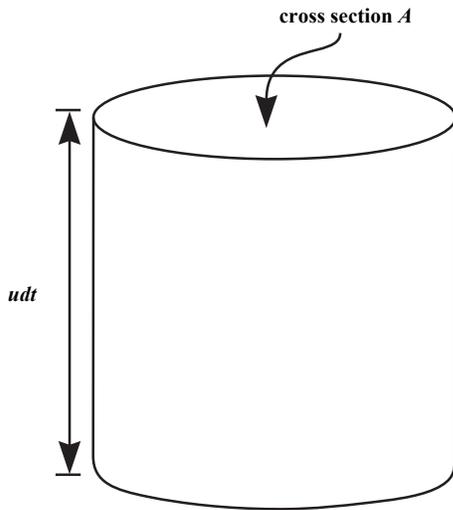


Fig. 1 Raindrops with terminal velocity u map out a volume of $Audt$

featured in the “The Last Word” of *New Scientist* (13 January 1995). Such discussions are usually only qualitative. However, the application of some simple physics should give us a more quantitative insight into this intriguing problem.

The simplest model is one in which the rain falls vertically downwards with no horizontal component to the velocity of the raindrops. We will also assume that the rainfall rate is constant with time.

If we now consider a circular area A into which raindrops are falling with a terminal velocity u , then in time dt the raindrops will map out a cylinder of height udt as shown in Fig. 1. Thus the volume of the cylinder is $Audt$.

If the total flux of rain (mass per unit area per unit time) is F , then the total mass of water crossing area A in time dt is $FAdt$. We can now define a raindrop density, ρ , as the mass of water per unit volume of space:

$$\rho = \frac{\text{total mass of water}}{\text{total volume}} = \frac{FAdt}{Audt} = \frac{F}{u} \quad (1)$$

There are two ways in which a person can encounter the raindrops. Firstly, the raindrops can fall directly on to the top surface of the person, and secondly, as the person moves they will run into those raindrops which are directly in front of them.

If we model a person as a cuboid with a top

surface area T and a frontal area S (Fig. 2), then in time dt the mass of rain falling on the top surface T is $FTdt$. If the person is moving with a constant velocity v , then in time dt they will map out a volume $Svdt$. So the mass of water hitting the front surface is $\rho Svdt$. Adding these two terms together gives a total mass of water encountered in time dt :

$$dM = (FT + \rho Sv)dt \quad (2)$$

The integration of this expression over time gives a relation between the mass of water encountered and the velocity at which the person travels for a given journey time τ . Assuming that the person travels with a constant velocity v , such that $\tau = l/v$ where l is the journey length, then

$$M = F(T/v + S/u)l \quad (3)$$

The flux, F , is simply a measure of the rainfall rate. If R is the rainfall rate expressed in millimetres per hour, then the flux, F , in kilograms per square metre per second, is just:

$$F = \frac{R\rho_1}{3.6 \times 10^6}$$

where ρ_1 is the density of water.

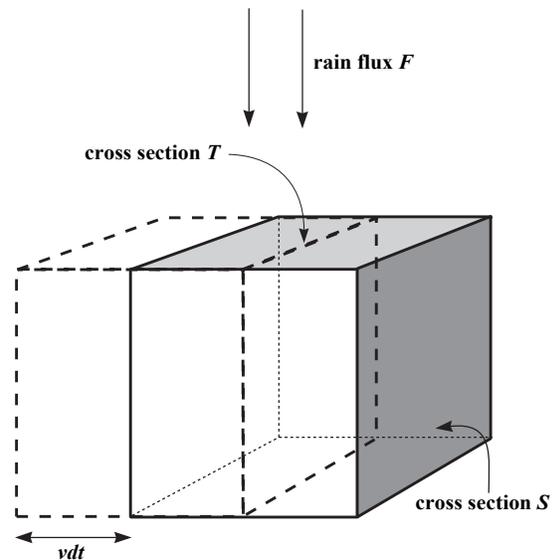


Fig. 2 Model of a person with top surface area T and front surface area S moving with a constant velocity v

So now for a given rainfall rate, R , we can determine the mass of water encountered, M , for a given velocity, v , for a journey length, l , using Eq. (3). The parameters T , S and u have been given typical values of 0.1 m^2 , 0.6m^2 and 5ms^{-1} respectively. Figure 3 shows this result plotted for a journey of 100 m for three rainfall rates: light rain (5 mm h^{-1}), the rate which is responsible for the main contribution to a year's total rainfall in a mid-latitude locality (10 mm h^{-1}), and heavy rain (15 mm h^{-1}).

Each of the curves tends to a constant value which represents the minimum mass of water which the person will hit (this value represents the fixed quantity of rain present in the person's path that will be encountered regardless of their speed). It can be seen that this minimum mass is reached quite rapidly for increasing velocity, suggesting that moving at speeds greater than about 0.5 ms^{-1} for light rain and about 2 ms^{-1} for heavy rain results in no significant reduction in the mass of water encountered. Given that walking pace is roughly $2\text{-}3 \text{ ms}^{-1}$, there is little point in running at about 6 or 7 ms^{-1} because we will get just as wet. Indeed, in order to justify sprinting at about 7 ms^{-1} a rainfall rate of

the order of 5 metres per hour would be required!

The rôle of the parameters in the problem can be understood more clearly if Eq. (3) is rewritten in the form:

$$M = \rho l S \left[\frac{uT}{vS} + 1 \right] \quad (4)$$

using Eq. (1). The effect that running at speed v has on how wet we get is contained in the dimensionless quantity uT/vS , which is the ratio of the amount of water collected on the top of our head to that collected by the front of our body. At very slow speeds, uT/vS is greater than 1 and more water is collected on our head than on our front. In this case, walking faster or running reduces our soaking. If we move more quickly, such that uT/vS is less than 1, the total amount of water collected is dominated by the rain which hits our front. This is determined by the factor $\rho l S$ in Eq. (4) which is a fixed quantity, independent of the speed at which we move.

When $uT/vS=1$, a person will get twice as wet as one who runs infinitely fast ($uT/vS = 0$). For typical values of T , S and u of 0.1 m^2

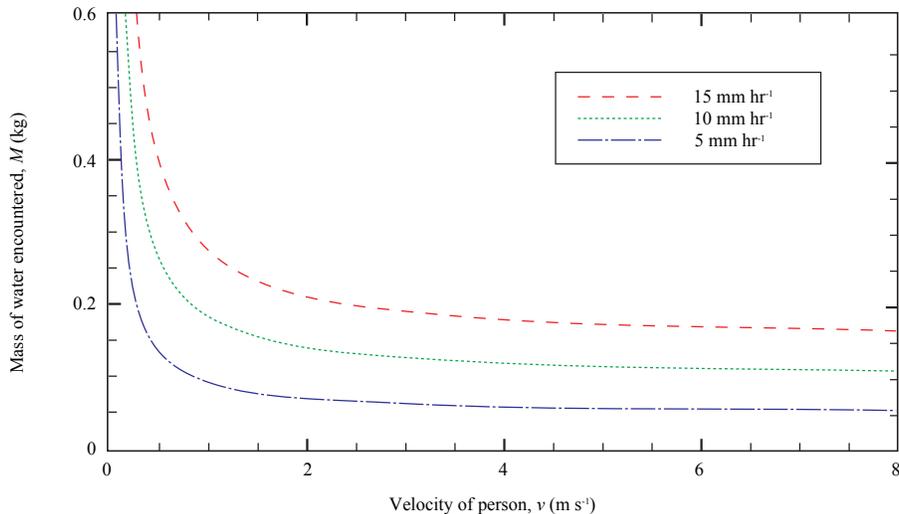


Fig. 3 The mass of rain encountered by a person moving with a velocity v for a journey of 100 m. Three curves are plotted corresponding to various rainfall rates.

0.6 m² and 5 ms⁻¹ respectively, this occurs when

$$v = \frac{5 \text{ ms}^{-1} \times 0.1 \text{ m}^2}{0.6 \text{ m}^2} \approx 0.8 \text{ ms}^{-1} \quad .$$

If this speed produces a water mass m , then doubling the speed to 1.6 ms⁻¹ gives a mass of 0.75 m . Doubling the speed again to 3.2 ms⁻¹ gives a mass of 0.62 m , while running at 6.4 ms⁻¹ gives a water mass of 0.56 m , a reduction of just 0.06 m . This latter 10 per cent reduction may be considered insignificant and not worth the 100 per cent increase in velocity. So a reasonable speed at which to travel would be about 3ms⁻¹ which is little more than walking pace!

These results are, perhaps, rather surprising

since they further raise the question: Why do we run when we are caught in the rain? Running can be justified when the rainfall rate is expected to increase, for example when it is just beginning to rain. In this situation, running reduces journey time and enables the destination to be reached before heavier rainfall sets in. If, on the other hand, the rain is particularly heavy, the rainfall rate might be expected to decrease within a short time. Now it might be advantageous to walk, giving enough time for the rainfall rate to decrease and allowing the journey to be completed in lighter rain. Yet, despite this, the majority of us can still be seen making a mad dash for shelter when caught unprepared in the rain.